

Aufgaben zu Wurzelkernen - Lösungen

1. $\sqrt{49a^4b^2} = 7a^2|b| \quad a, b \in \mathbb{R}$

$$\sqrt{(-b)^2} = |b| \quad b \in \mathbb{R}$$

$$\sqrt{-b}^2 = -b = |b| \quad b \in \mathbb{R}_0^-$$

$$\sqrt{(1-2x)^2} = |1-2x| \quad x \in \mathbb{R}$$

$$\sqrt{(x-y)^2} = |x-y| \quad x, y \in \mathbb{R}$$

$$\sqrt{x^2+y^2} \quad x, y \in \mathbb{R}$$

$$\sqrt{x^2 \cdot y^2} = |x| \cdot |y| \quad x, y \in \mathbb{R}$$

2. a) $\sqrt{x^2} = -x$ wahr für alle $x \in \mathbb{R}_0^-$

$$\sqrt{(x-1)^2} = x-1 \quad \text{wahr für alle } x \in [1; \infty[\quad (x-1 \geq 0)$$

b) $\sqrt{169} \cdot x + \sqrt{169 \cdot x^2} =$ Der Term hat den
 $13x + 13|x| =$ Wert 0 für $x \in \mathbb{R}_0^-$.
 $13(x + |x|)$

3. a) $(1 - \sqrt{3})(1 + \sqrt{3}) \stackrel{3.BF}{=} 1^2 - \sqrt{3}^2 = 1 - 3 = \underline{\underline{-2}}$

b) $(\sqrt{2} - 3\sqrt{2})^2 = (-2\sqrt{2})^2 = 4 \cdot 2 = \underline{\underline{8}}$

c) $\sqrt{3} \left(\frac{1}{6}\sqrt{12} - 3 \cdot \sqrt{\frac{1}{27}} \right) \stackrel{D\text{-Gesetz}}{=} \frac{1}{6} \cdot \sqrt{36} - 3\sqrt{\frac{1}{9}} = 1 - 1 = \underline{\underline{0}}$

d) $(2\sqrt{108} - 7\sqrt{54}) : \sqrt{27} \stackrel{2.BF}{=} 2 \cdot \sqrt{4} - 7\sqrt{2} = \underline{\underline{4 - 7\sqrt{2}}}$

e) $(\sqrt{2} - \sqrt{18})^2 \stackrel{2.BF}{=} \sqrt{2}^2 - 2 \cdot \sqrt{2} \cdot \sqrt{18} + \sqrt{18}^2 = 2 - 12 + 18 = \underline{\underline{8}}$

oder: $= (\sqrt{2} - 3\sqrt{2})^2 = \underline{\underline{8}} \quad (\text{s. 3b})$

f) $2\sqrt{7} - 3(1 - \sqrt{28}) = 2\sqrt{7} - 3(1 - 2\sqrt{7}) = \underline{\underline{4\sqrt{7} - 3}}$

g) $3\sqrt{63} + 6\sqrt{72} - 4\sqrt{28} - 17\sqrt{8} = 9\sqrt{7} + 36\sqrt{2} - 8\sqrt{7} - 34\sqrt{2}$
 $= \underline{\underline{\sqrt{7} + 2\sqrt{2}}}$

h) $\sqrt{\frac{a}{2-a}} \cdot \sqrt{2a-a^2} = \sqrt{\frac{a}{2-a} \cdot a \cdot (2-a)} = \sqrt{a^2} = \underline{\underline{a}} \quad (a \in [0, 2])$

i) $\sqrt{\frac{a}{3b}} : \sqrt{\frac{b^3}{27a}} = \sqrt{\frac{a}{3b} \cdot \frac{27a}{b^3}} = \sqrt{\frac{9a^2}{b^4}} = \underline{\underline{\frac{3a}{b^2}}}$

j) $\sqrt{xy^2} \cdot \sqrt{\frac{8}{y^2}} - \sqrt{2x} = 2\sqrt{2x} - \sqrt{2x} = \underline{\underline{\sqrt{2x}}}$

$$\sqrt{xy^2} \cdot \sqrt{\frac{8}{y^2}} - 2\sqrt{x} = 2\sqrt{2x} - 2\sqrt{x} = 2\sqrt{x}(\sqrt{2} - 1)$$

$$\sqrt{xy^2} \cdot \sqrt{\frac{8}{y^2}} - x\sqrt{2} = 2\sqrt{2x} - x\sqrt{2} = \underline{\underline{\sqrt{2}(2\sqrt{x} - x)}}$$

$$4. a) \frac{1}{\sqrt{a}} = \frac{1 \cdot \sqrt{a}}{\sqrt{a} \cdot \sqrt{a}} = \frac{\sqrt{a}}{a}$$

$$b) \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\underline{2}} \quad (\text{s. 4a})$$

$$\frac{5}{2\sqrt{3}} = \frac{5 \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} = \frac{5\sqrt{3}}{6} = \underline{\underline{\frac{5}{6}\sqrt{3}}}$$

$$\frac{25}{\sqrt{125}} = \frac{25}{5\sqrt{5}} \stackrel{\text{Kürzen!}}{=} \frac{5 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{5\sqrt{5}}{5} = \underline{\underline{\sqrt{5}}}$$

$$\frac{\sqrt{x} - \sqrt{y}}{\sqrt{xy}} = \frac{(\sqrt{x} - \sqrt{y}) \cdot \sqrt{xy}}{\sqrt{xy} \cdot \sqrt{xy}} = \frac{x\sqrt{y} - y\sqrt{x}}{xy} = \underline{\underline{\frac{\sqrt{y}}{y} - \frac{\sqrt{x}}{x}}}$$